D. Kellogg; "Note on the potential and the antipotential group of a given group," by G. A. Miller; "The equation of Picard-Fuchs for an algebraic surface with arbitrary singularities," by S. Lefschetz; Review of Manning's Geometry of Four Dimensions, by J. L. Coolidge; "Shorter Notices"; Schröder's Entwicklung des mathematischen Unterrichts an den höheren Mädchenschulen Deutschlands, by E. B. Cowley; de Montessus and d'Adhémar's Calcul numérique and Dickson's Elementary Theory of Equations, by R. D. Carmichael; Smith's Teaching of Geometry and Smith and Mikami's History of Japanese Mathematics, by J. V. McKelvey; Study's Die realistische Weltansicht und die Lehre vom Raume and Jordan and Fiedler's Contribution à l'Etude des Courbes convexes fermées et de certaines Courbes qui s'y rattachent, by Arnold Emch; Mrs. Gifford's Natural Sines to Every Second of Arc, and Eight Places of Decimals, by D. E. Smith; Cobb's Applied Mathematics, by E. B. Lytle; von Sanden's Praktische Analysis and Hjelmslev's Darstellende Geometrie, by Virgil Snyder; "Notes"; and "New Publications."

The March number of the *Bulletin* contains: Report of the twenty-first annual meeting of the society, by F. N. Cole; Report of the winter meeting of the society at Chicago, by H. E. Slaught; "The structure of the ether," by Harry Bateman; "Shorter Notices": Killing and Hovestadt's Handbuch des mathematischen Unterrichts, Band II, by D. D. Leib; Cahen's Théorie des Nombres, Tome premier, and Darboux's Théorie générale des Surfaces, première Partie, by T. H. Gronwall; "Notes"; and "New Publications."

SPECIAL ARTICLES

INTERPOLATION AS A MEANS OF APPROXIMATION TO THE GAMMA FUNCTION FOR HIGH VALUES OF n [1]

Various approximations to the value of $\Gamma(n)$ when n is large have been suggested by different workers and are in every-day use. In actual statistical practise the one which has appealed to the writer as most satisfactory, having regard to ease of calculation and degree of accuracy of result, is that of Forsyth,[2] which is

$$\Gamma(n+1) = \sqrt{2\pi}\left(\frac{\sqrt{n^2+n+\frac{1}{6}}}{e}\right)^{n-\frac{1}{2}}.$$

This is in error (in defect) in the proportion of $1/240n^3$.

It lately occurred to me that possibly a further saving of labor in computation, without loss of accuracy, could be made by interpolating in a table of log $\underline{|n}$ to get log $\Gamma(n)$. Tables of the sums of the logarithms of the natural numbers have recently been made readily available to statistical workers from different sources.[3] Such tables all proceed, of course, by integral steps of the argument n.

The question then is to determine what the order of magnitude of the error will be if one interpolates from such a table proceeding by integral steps, in order to determine $\Gamma(n)$. The relation

$$\Gamma(n+1) = \underline{|n} \qquad \text{(i)}$$

is exact when n is an integer. How great is the inequality when n is not integral but fairly large?

To test this matter I asked Mr. John Rice Miner, the staff computer of the laboratory, to carry through the computations for a short series of representative values of n. This he has done, with the results set forth in Table I., for which I am greatly obliged. It should be said that in all the computations seven-place logarithms only have been used. The first column, headed "exact value," gives the result obtained by using the value of log $\Gamma(x)$ for $x = 1.123$ from Legendre's tables, and then summing the logarithms up to $n-1$ for each desired value. This is the usual process, depending on the relation

[1] Papers from the Biological Laboratory of the Maine Agricultural Experiment Station, No. 80.

[2] Forsyth, Brit. Assoc. Rept. for 1883, p. 47.

[3] *Cf.* Pearl and McPheters, *Amer. Nat.*, Vol. XLV., 1911, p. 756. More recently a longer table of sums of logarithms has been published in Pearson's "Tables for Statisticians and Biometricians," Cambridge, 1914.

$$\Gamma(n+1) = n\,\Gamma(n) = n(n-1)(n-2) \\ \ldots (n-r)\,\Gamma(n-r). \quad \text{(ii)}$$

It becomes an exceedingly tedious operation when n has a value of over, say, 20. In calling this the "exact" value in the table the intention is merely to convey the idea that the only approximation involved is that incident upon the use of 7-place logarithms, the process *per se* being an exact one. The fourth and fifth columns of the table give the results obtained by using the values of log $\lfloor n$, their first second and third differences, in the usual advancing difference interpolation formula

$$u_{x+n} = u_x + n\,\Delta\,u_x \\ + {}_nC_2\,\Delta^2\,u_x + {}_nC_3\,\Delta^3\,u_x \cdots. \quad \text{(iii)}$$

TABLE I

Values of log Γ (n) by Different Methods

n	Exact Value	Forsyth's Approximation	Interpolation Using Δ^2	Interpolation Using Δ^3
5.123	1.4613860	1.4613679	1.4619138	1.4615009
15.123	11.0834931	11.0834916	11.0835559	11.0834985
25.123	23.9637108	23.9637096	23.9637336	23.9637119
35.123	38.6594135	38.6594126	38.6594251	38.6594138
75.133	107.7498704	107.7498692	107.7498727	107.7492870

From this table it is evident that the interpolation method, when third differences are used, gives values slightly better than those by Forsyth's method when $n \gtreqless 25$. For $n = 75$ or more the interpolation method using only second differences gives an approximation sufficiently close for all practical statistical purposes. As to the labor involved, there is no great amount of choice between Forsyth's and the interpolation method, but on the whole there appears to be a distinct, if small, advantage in favor of the interpolation.

RAYMOND PEARL

THE GEOLOGICAL SOCIETY OF AMERICA

THE twenty-seventh annual meeting of the Geological Society of America was held at the Academy of Natural Sciences, Philadelphia, December 29–31, 1914, under the presidency of Dr. George F. Becker, of the United States Geological Survey, Washington, D. C. On account of Dr. Becker's enforced absence through illness, the sessions were presided over by Vice-presidents Waldemar Lindgren and Horace B. Patton. In attendance there were registered 117 Fellows of the Society and the number of students and others, including members of the American Association for the Advancement of Science who were present at the sessions, swelled the attendance to more than 200, making this one of the most largely attended meetings in the history of the society.

At the first general session of the society Dr. Samuel G. Dixon, president of the Academy of Natural Sciences, welcomed the visiting geologists and paleontologists, making them feel very much at home as the guests of the historic academy.

The report of the council, as submitted in print, showed that the present enrollment of the society is 363, aside from the 19 new fellows elected at the meeting but who had not yet qualified. During the year 1914 the society lost five fellows by death: Alfred E. Barlow, Albert S. Bickmore, Horace C. Hovey, A. B. Wilmott and Newton H. Winchell; and three correspondents: H. Rosenbusch, Eduard Suess and Th. Tschernyschew. The treasurer's report showed that the society was in a flourishing condition financially and the editor's report indicated an unusual activity in publication during the past year.

The papers presented in the three general sessions of the society were as follows:

Relation of Bacteria to Deposition of Calcium Carbonate: KARL F. KELLERMAN.

At the suggestion of Dr. T. Wayland Vaughan, bacterial studies of water and bottom mud from the Great Salt Lake, and sea water and bottom deposits from the vicinity of Florida and the Bahamas were undertaken in the hope of supplementing the work of Vaughan,[1] of Drew[2] and of Dole[3] in regard to the probable agencies concerned in the precipitation of calcium carbonate and the formation of oolites.

It has been possible to form calcium carbonate by the action of bacteria on various soluble salts of calcium both in natural waters and in synthetic mixtures. The most important natural precipita-

[1] T. Wayland Vaughan, *Bull. Geol. Soc. Am.*, Vol. 25, No. 1, p. 59, March, 1914. Also Publication No. 182, Carnegie Inst. of Washington, pp. 49–67.

[2] G. H. Drew, Publication No. 182, Carnegie Inst. of Washington, pp. 49–67.

[3] R. B. Dole, Publication No. 182, Carnegie Inst. of Washington, pp. 69–78.